Model-Based Analysis of Two Fighting Worms

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Abstract

Self-replicating malicious codes (worms) are striking the Internet vigorously. A particularly sophisticated recent introduction is the “killer” worm (also called counter-worm or “predator” worm). The goal of this research is to explore the interaction dynamics between a worm (prey) and an antagonistic worm (predator), using mathematical modeling. This paper models several interesting combat scenarios of two fighting worms, including the effect of antivirus on the system behavior. There are few novel findings of our enhanced model, such as the prediction of oscillatory behavior of interacting worms population conforming to existing biological systems.

1. Introduction

Computer viruses are increasingly becoming a major source of productivity drain for internet operations. A particularly sophisticated recent introduction is the killer worm (also called counter-worm, predator worm, or good will mobile code). This is a new phenomenon that has made headlines recently. These worms are out there fighting malicious codes (Code-Red, MS-Blast, and Sasser) spread by rival virus writer groups.

There is an interesting digital culture which helps the emergence of these predator worms. For example, one worm’s authors fight another group to expand their peer-to-peer networks, which are later used to launch new worms, generate Denial of Service attacks, or circulate spam anonymously. In addition, a predator worm may spread through a flaw or backdoor of another worm. While, predator worms can be malicious, they also can be the necessary proactive countermeasure to fight zero-day worms.

The goal of this research is to mathematically model the behavior of combating worms. This paper models prey-predator dynamics for different interesting combat scenarios. For each scenario we present a mathematical model that is based on Lotka-Volterra equations and then present the corresponding analysis using numerical solutions.

1.1 Related work

While modeling worms is not totally new, there’s only very few in literature about killer virus (predator worm). Two papers are in the same line as our work. Toyoizumi and Kara used Lotka-Voltera equations to model and analyze the interaction between predator worms and traditional worms [1]. They define predators as “good will mobile codes” that kill malicious viruses. Also, they discuss how to minimize predator population without losing their effectiveness. Nicol and Liljenstam define active defenses, as techniques that “take the battle to the worm” [4]. They model four active defenses two of which are predator worms. They also define some effectiveness metrics such as the number of protected hosts, total consumed bandwidth, and peak scanning rate.

Staniford was the first to attempt to model random scanning Internet worms [5]. His model is a quantitative theory that explains Code-Red spread. The theoretical data generated by his equation fairly matched with available Code-Red data. Later Zou et al provided an enhanced model of Code-Red that considers the effects worm countermeasures and routers congestion [6]. They base their model on Kermack-Mckendrick and their simulations and numerical solutions better match Code-Red data.
2. Model Basis

2.1 Virus Types

Although the terminologies have not been firmly established in literature here we use the term virus to relate to the superset of self-replicating malicious codes. A worm is a subset of viruses that is network-aware (use network protocols and parameters to spread). Worms can be fully-automated (use port-scanning) or human-dependent (spread through email.)

Traditional ways to defend against worms—called defensive techniques (or countermeasures) are based on preventing, detecting, and cleaning virus infections. These countermeasures include Antivirus and System patches. While Antivirus programs can detect and clean worms’ infections, System patches cannot remove a virus instead it can fix an existing security hole and thus prevent worm infection. System patches are made available by operating system authors.

Most predators spread by exclusively penetrating already infected machines, called infection-driven predator worms. However, some predators attack both infected and clean machines, called Vulnerability-driven predator worms. A predator worm that actively scans for prey-infected machine is called active-spreading predators. On the other hand, some predator worms don’t search for a prey worm, instead the prey fall in trap once it unknowingly scans a predator-infected machine. Such a predator that depends on prey to scanning, is called passive-spreading predator, e.g. CR-Clean. Figure 2.1 shows that predator worms are first classified according to their victim infection state and then classified further according to their scanning technique.

As shown in Figure 2.1, prey worms can be patching or non-patching. Prey worms may protect themselves from their predators by closing the security hole through which they penetrated, thus preventing predator from getting in. We call such prey worms a patching worm otherwise they are non-patching prey worms.

Worms that can attack an infected machine, wipe the existing worm, and takeover that machine are called predator worms, e.g. Code-Green, Welch, and Netsky. On the other hand, prey worms are the victims of predator worms, e.g. MS-Blast, Bagle, and Sasser. Figure 2.1 explains the classification. Internet worms according to their predatory role.

2.2 Environment

We assume The Internet size is fixed during any infection cycle. Thus, total number of machines is $M$ which is constant. Any machine can be either susceptible to an infection by some worm (called vulnerable) or immune (called removed). Vulnerable machines can be penetrated by a worm, and once infected they spread the infection on their own. Removed machines cannot be infected by a worm for
some reason; e.g. the worm doesn't run on that machine's platform or the machine doesn't have the related security flaw. If the number of vulnerable machines is $S$, and number of immune machines is $R$, then $S + R = M$ is the total number of machines. Figure 2.2 shows the two main sets set-S and set-R. Usually, vulnerable and removed machines don’t switch back and forth. However, in some cases a vulnerable machine may become immune; e.g. when and operating system patch is applied such that the related security flaw is fixed.

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A vulnerable machine that is infected by a worm is called infectious. All other vulnerable machines that are not compromised are in the clean machines set (set-n) of size $n(t)$. Machines can change their state from clean to infectious, or infectious to clean. We assume that an infectious machine is infected by only one of two worms: a prey or a predator worm. Infectious machines that are infected by a prey worm (worm-x) are called set-x, which has cardinality of $x(t)$. Machines that are infected by predator worm (worm-y) form set-y with of size $y(t)$. Figure 2.2 shows the two infectious sets and their relation to the clean set. Machines in set-x can change state and move into set-y. The cardinalities of set-n, set-x, and set-y, are variable functions of time, where the total sum $S = n(t) + x(t) + y(t)$ is the size of vulnerable machines set.

3. Scenario-1: Prey, Predator Model

In the basic scenario, two combating worms (a prey and a predator) spread over a network. Worm-x is a traditional prey worm, which spreads by infecting clean machines. Worm-y is infection-driven predator worm that can spread only by taking over machines infected by worm-x. The size of worm-x population at anytime is $x(t)$ while size of worm-y population is $y(t)$. Figure 3.1 describes the interaction between the different sets in this scenario. Directed links signifies the transition rate of members between two sets.

Set-x size increases at rate proportional to both the size of set-x and set-n. In other words, at anytime the increase in the number of machines infected by worm-x depends on the number of already worm-x machines and the number of existing clean machines. On the other hand, any encounter between worm-x and worm-y instances will result in an increase in worm-y population on count of worm-x population. Thus, set-y size increase at rate proportional to the number of worm-x and worm-y infected machines.

The infection rate of worm-x is the first derivative of $x(t)$. The same applies to worm-y and clean machines change rate. The dynamic of the system are described by equations 3.1, 3.2, 3.3, and 3.4.

$$\frac{dx}{dt} = a_{xn} - b_{xy}$$  \hspace{1cm} (3.1)

$$\frac{dy}{dt} = b_{xy}$$  \hspace{1cm} (3.2)

$$\frac{dn}{dt} = -a_{xn}$$  \hspace{1cm} (3.3)

$$x(0) = x_0, y(0) = y_0, n(0) = n_0$$  \hspace{1cm} (3.4)
Both $a$ and $b$ are positive parameters that depend on worms’ scanning rate and network size. Below, we discuss the derivation of $a$ and $b$ values.

### 3.1 Parameters Derivation

Let worm-x scanning rate be $r$, where $r$ is the number of unique scans generated by the worm per a unit of time. Thus, the total number of scans by all members in set-x is $rx$. Since $R+x(t)+y(t)+n(t)=M$, the value of $rx$ is the sum of all scans by worm-x of all machine sets, as in equation 3.5

$$rx = \frac{rxR + rx^2 + rxy + rxn}{M} \quad (3.5)$$

If each time that worm-x scans a clean machine results in a new infection, then parameter $a$ is given by equation 3.6

$$a = \frac{r}{M} \quad (3.6)$$

Likewise, if every encounter between $y$-worm and worm-x infected machine results in a takeover by worm-$y$, then parameter $b$ is given by equation 3.7

$$b = \frac{r}{M} \quad (3.7)$$

The previous discussion applies to passive-spreading predator. On the other hand, an active-spreading predator does its own scanning in order to find worm-x infected machines. If we assume that worm-$y$ has scanning rate be $v$, the total number of scanning by members in set-$y$ is $vy$ satisfies equation 3.8

$$vy = \frac{vyR + vyy + vy^2 + vyn}{M} \quad (3.8)$$

Since encounters between worm-x and worm-$y$ result from both scans by worm-x and worm-$y$. The parameter $b$ can be described by equation 3.9

$$b = \frac{v + r}{M} \quad (3.9)$$

### 3.2 Analysis

We used numerical solution to solve the equation system described in 3.1, 3.2, 3.3, and 3.4. We used Maple to draw the curves in figure 3.2 and 3.3. Multiple curves in red for $x(t)$ and in green for $y(t)$ are plotted for different values of $a : b$.

![Figure 3.2](image1.png)

**Figure 3.2.** $a. M=10, M=3000000, x_0=100, y_0=1, n_0=1000000.$

![Figure 3.3](image2.png)

**Figure 3.3.** $a. M=10, M=3000000, x_0=100, y_0=1, n_0=1000000.$

The general behavior described here shows that initially worm-x increase exponentially as it would without worm-$y$ existence. Worm-$y$ increase proportional to increase in worm-x populations. The increase in worm-$y$ population results in decrease in worm-x population, as worm-$y$ takeover worm-x machines. The $x(t)$ curve reaches its maximum when it infects all vulnerable machines (figure 3.2) or when worm-$y$ is large enough to consume more worm-x machines than can worm-x reproduce (figure 3.3). Curve $y(t)$ continues to increase until it uses up all available worm-x members, where it hits its maximum and freeze thereafter. The system reaches steady state when both infection rates are zero. This occurs when
all worm-x infected machines are re-infected by worm-y \((x(t)\) is zero).

In figure 3.2 \(\max(x) = \max(y) = S\). In other words the maximum value of the curves is size of vulnerable population. We name this condition as Prey-outbreak condition since it occurs as result of faster growth in prey population than predator population \((b \leq a)\)

In figure 3.3 \(\max(x) \leq \max(y)\). This condition is called prey-cutback condition, which occurs when the predator population is growing faster than the prey \((b > a)\).

4. **Scenario-2: Prey, Vulnerability-Driven Predator Model**

We expand the basic by considering vulnerability-driven type of predator, where worm-y can infect both clean and worm-x infected machines. Figure 4.1 describes the transitions between the machines sets. Worm-x increases as in the basic scenario. However, worm-y increases by targeting clean machines at rate \(cyn\) \((c\) is positive) in addition to infecting worm-x machines. The system dynamics can be described in equations 4.1, 4.2, 4.3, and 4.4.

\[
\frac{dx}{dt} = axn - bxy \quad (4.1)
\]

\[
\frac{dy}{dt} = cyn + bxy \quad (4.2)
\]

\[
\frac{dn}{dt} = -axn - cyn \quad (4.3)
\]

\[
x(0) = x_0, \ y(0) = y_0, \ n(0) = n_0 \quad (4.4)
\]

Parameter \(a\) and \(b\) values are as derived in previous section. The value of \(c\) which depends on both worm-y scanning rate and network size is given in equation 4.5

\[
c = \frac{v}{M} \quad (4.5)
\]

In case of vulnerability-driven predator \((c > 0)\) the predator has more than one way to spread and thus isn't totally dependent on the prey population. Any increase in prey population will increase the predator population and increasing the predator population will decrease prey population. However, a decrease in the prey population will not lead to a decrease in the predator's population.

5. **Scenario-3: Prey, Predator, and Antivirus Model**

Worm-x and worm-y are prey and predator worms that are competing over an environment. Worm-y is vulnerability-driven predator. Some machines on the network run antivirus software that can detect and clean both worms' infections. This scenario is
analogous to harvesting (spraying, or fishing) phenomena in biological systems, where some third-party eliminates members of both combating populations. We assume that as people become aware of an epidemic, they start to install or update antivirus software at increasing rate.

We assume that the number of machines with antivirus update to be an increasing function of time. The functions $z_x(t)$ and $z_y(t)$ are the fraction of worm-x and worm-y infected machines, respectively, that are cleaned by the antivirus software at anytime. We define $z_x(t)$ and $z_y(t)$ in equations 4.1 and 4.2.

The constants $d_x$ and $d_y$ are fraction numbers that determines the antivirus effectiveness.

$$z_y(t) = d_y t / (t + 1)$$

$$z_x(t) = d_x t / (t + 1)$$

$$\frac{dy}{dt} = cyn + bxy - yz_y \quad \text{(5.4)}$$

$$\frac{dn}{dt} = -axn - cyn + xz_x + yz_y \quad \text{(5.5)}$$

$$x(0) = x_0, y(0) = y_0, n(0) = n_0 \quad \text{(5.6)}$$

Figure 5.1 describes the transition of members between machines’ sets as a result of the two worms and antivirus reactions. Worm-x increase on count of clean machines set (set-n) at rate $axn$. Meanwhile, set-n gains worm-x machines back at rate $xz_x$, once cleaned by the antivirus. On the other hand, worm-y increase on count of both clean and worm-x machines at rate $cyn + bxy$. In contrary of all previous scenarios, set-y decreases at rate $yz_y(t)$, as result of antivirus effect. The system behavior is described by equations 5.3, 5.4, 5.5, and 5.6

$$\frac{dx}{dt} = axn - bxy - xz_x \quad \text{(5.3)}$$

Figure 5.2 shows a new type of behavior, both curves $x(t)$ and $y(t)$ oscillate for a while as they gradually become constant lines. This phenomenon is a result of introducing the antivirus effect, which kills predators as well as prey infections. Originally, the increase in predators population causes degrade in prey population, and this is what is initially happening in this case. However, as the antivirus cleans some predator infections causing its population to drop, more prey infections will have chance to survive, and thus prey population increases again. Increasing prey population results in increasing predator population. However the second peak is lower than the first once since the antivirus is continuously reducing both populations. This periodical behavior repeats itself each time with lower maximum values. The oscillation turns into straight lines with some vibration, which eventually diminishes, resulting into two constant lines. At this stage the system reaches its steady state or equilibrium point.

6. Conclusion & Future Work
In this paper we have presented several scenarios of virus-virus warfare. We classify worm types according to their predatory characteristics. We study and analyze the prey and predator interaction, and investigate the related parameters' values. We study several advanced scenarios, including antivirus effect on prey-predator system. Since the beginning of this work coincidentally several ware-fare has been reported in real Internet. However, we must warn this work does not model the specific warfare.

There are actually additional scenarios which can be potentially modeled. One example is **Cascade Chain Worms (Wave Worm).** Many worms have more than one version. The new versions are meant to update the old ones. However, existence of old versions can have positive or negative effect of the spread of the new version. Our current model considers the number infected machines to be the worm population size. This is true as long as each machine has only single infection. In the future we will extend our work to study the **Multi-Infection machine scenario.** Up to date, all existing models, including those in this paper, are based on random network model. In reality, the Internet is a scale-free network [9], which can help in the spread of worms' vaccines [8].

7. References


